The Environmental Impact of Internet Regulation *

Jean-Christophe POUDOU† Wilfried SAND-ZANTMAN ‡

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Abstract

We address the need to regulate Internet infrastructure usage to take into account environmental externalities. We model the interactions between an internet service provider and some content providers in settings where the former chooses the network size and the latter influence congestion on the network. We then discuss how different regulatory frameworks (Net Neutrality, Laissez-Faire, Price Regulation) impact both the economic efficiency and the environmental performance. In particular, we highlight that inducing more efficiency effort from the content providers may either improve or worsen the final environmental outcome, and provide some conditions for one case or the other to prevail.

Keywords Carbon Emission, Environmental Externalities, Investment, Net Neutrality, Regulation

IEL Classification D4; L1; L51; L86

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[†]MRE, University of Montpellier. E-mail: jean-christophe.poudou@umontpellier.fr ‡ESSEC Business School and Thema. Email: sand@essec.edu

1 Introduction

In recent decades, the telecom and digital sector has witnessed growth that is unprecedented in economic history. A whole new world of social media, online advertising and online sales has emerged, and this world has relied on investments by some actors of the traditional sectors, in particular the construction and telecom industries. However, the increase in energy prices and the negative environmental impact of digital industries have put into question the sustainability of the current trend. Indeed, the digital sector is said to account for 3 to 4% of worldwide greenhouse gas output with annual growth of approximately 8% (see [ARCEP, 2020\)](#page-30-0) and [Obringer et al.](#page-32-0) [\(2021\)](#page-32-0) consider as overlooked the carbon footprint of internet usage, assessed to lie between 28 to 63 gCO2 equivalent per gigabit. For instance according to [Stephens et al.](#page-32-1) [\(2021\)](#page-32-1), the European average carbon emissions per day of video streaming for the year 2020 is estimated to be 168 gCO2e, these are the emissions of driving a standard car for 1km.

Context The objective of this paper is to address the need to regulate the use of infrastructures to mitigate the environmental cost of internet usage. This idea seems to be nowadays on the political agenda in the European Union. According to the European Green Deal digital technologies may ensure to save more energy than they consume, (see [European Commission, 2022\)](#page-31-0). Recently, Commissioner Thierry Breton insisted on the urgency of developing "an industrial policy that manages the green and digital transition while preserving our competitiveness". In the same vein, a report for the European Commission, DG Environment, (see [Liu et al., 2019\)](#page-32-2) points out that the regulation of the data economy should be subjected to scrutiny according to the environmental implications of existing proposals, for instance in mitigating data-monopolies.

In our work, we focus on the negative aspects of the environment of ICT and Internet usage. Of course, we are fully aware that there are environmental opportunities related to digital transformation (see for instance [Liu et al.](#page-32-2) [\(2019\)](#page-32-2)). In our analysis, we do not advocate, nor do we give results that point in the direction of a "decline" in digital usage. We simply analyze regulatory reforms such that consumers are still using Internet services but with the industry being disciplined to use

infrastructures in a more environmental-friendly way.

To date, digital debates concerning infrastructures have mainly focused on the net neutrality issue, i.e., the extent to which content providers (mostly social media but, more globally, businesses operating on the Internet) should pay Internet Service Providers (ISPs) to reach consumers. On the one hand, most ISP have claimed that content providers have to pay to use their infrastructures to provide them the right incentives to invest in capacity. On the other hand, content providers have claimed that ISPs are already being paid by their subscribers and that any additional fee would only limit the variety of content available, at the expense of consumers. In this debate, most of the arguments are assessed on the impact (positive or negative) on the total level of investment. The environmental crisis should lead us to consider how to make the best use of the network as least as much as the best way to increase total capacity.

The reason for considering the total capacity of the network as a key variable is threefold. First, the capacity of the network puts an upper bound on the use of the internet; that is, the flow of data, by all undertakings. Therefore, if the actions of the agents generate environmental damages, they are related to network capacity. Second, building extra capacity directly induces some direct environmental costs (e.g., raw material, use of energy). Third, the management of this capacity is very costly. According to a recent report (see France Stratégie, 2020), 75% of Telco electricity consumption is driven by network operations, so limiting its growth is a direct way to limit this consumption (and the associated externalities). However, for a given size of the network, how it is used no longer depends on telecom operators but rather on content providers and consumers. Concerning content providers, many of them can decide to use the network in a more or less effective way. Indeed, they can use data compression techniques or store the data closer to consumers in order to minimize their use of network capacity. Regarding consumers, they can reduce the intensity of internet usage either by altering the video parameters or their time allocation across websites. But neither content providers nor consumers directly benefit from these actions. Therefore, this change of behavior will only happen if the agents are given the proper incentives either by the terms of the contract they sign with the ISPs or by adequate regulation. In this article, we discuss the possible solutions to the presence of these externalities.

More precisely, we develop the idea, in line with the proposals of some regulators (see for example [ARCEP, 2020\)](#page-30-0) that more should be done to incentivize the content providers (CP). Indeed, the amount of data they generate is an important driver of the capacity choice by the ISP, and therefore of the carbon footprint of the industry. To that end, we develop a model in which an ISP allows consumers to access the service of some ad-financed CPs. For this service to be provided, the ISP must choose the network size (or capacity) that has both a private cost and an environmental cost. But the necessary size of the network depends on the characteristics and technology of the CPs. In line with the real world, we consider different types of CP that differ according to their impact on the network, i.e., how much capacity they need, but also on their ability to reduce the needed capacity. Some CPs are capacity-intensive and can, at a small cost, significantly reduce their traffic load, whereas other CPs require less capacity but can barely further reduce their traffic load. In this setting, the level of consumption, the actions of the CPs, and size of the network should take into account the impact on consumers, the ad revenues generated by the CP, and the cost of the capacity, both for the ISP and the environment.

Main results We first look at a situation in which the only monetary transaction is between consumers and the ISP. This case, called net neutrality, suffers from many inefficiencies. First, CPs are never incentivized to reduce the load they generate on the network. Second, the ISP does not take into account the revenues generated by the CP or the negative environmental externalities.

Then, we compare to the laissez-faire situation in which the ISP can freely charge CPs. If the ISP uses a fixed price per unit of traffic, this will generally results in higher investment levels and therefore a negative environmental impact. Indeed, the ISP can capture the revenues generated by the CPs, so its marginal gain from expanding the network size is increased. Nevertheless, we show that laissez-faire could lead to reducing the network size and therefore the environmental cost of the sector in two cases. First, if the CPs differ in the revenues they generate, then the ISP can choose to exclude some CPs, and the equilibrium size of the network decreases. Second, and more interestingly for us, when the ISP can charge the CPs for the congestion they create, it may give them some incentives to reduce the congestion on the network, although its incentives to do so are reduced compared to that of a benevolent planner.

However, even in this case, the prospect of capturing ad revenues may drive the ISP to choose a larger network size than in the net neutrality case. In our setting, this ISP choice are driven by two main forces. First, when congestion decreases, for a given consumer level of usage, the capacity need is decreased (a pure *congestion-based e*ff*ect*). Second, a lower congestion cost increases the incentives to propose higher usage levels to consumers (a *consumption-based e*ff*ect*). When consumer marginal utility is quite sensitive to changes in quantity, which could be justified by a standard ratchet effect in consumption, the first effect dominates, and decreasing congestion leads the ISP to decrease its capacity investment.

Finally, we consider the possibility that a regulator sets some congestion-based prices for the CPs. This new regulatory tool gives the right incentives to the CPs without directly affecting the ISP's incentives to increase the size of the network. Moreover, we see how this price level should be optimally set to take into account not only consumer surplus and the cost of building the network but also the environmental externalities.

Literature This paper is related to the net neutrality debate and, therefore, to the literature that addresses the impact of allowing or preventing the ISP to discriminate, in price or quality, among the content providers for accessing their subscribers. In this debate, net neutrality has been defined either as the ban on prioritization see, for example, [Choi and Kim](#page-31-2) [\(2010\)](#page-31-2), [Bourreau, Kourandi, and Valletti](#page-30-1) [\(2015\)](#page-30-1) or [Gautier and Somogyi](#page-31-3) [\(2020\)](#page-31-3) —or as a restriction of the way ISPs can charge CPs. We take the second approach, which was first developed by [Economides and Tåg](#page-31-4) [\(2012\)](#page-31-4). Whereas this article emphasizes the two-sided nature of the industry, we are more concerned by the impact on investment and congestion of regulation. In particular, we develop the idea that the need for investment is the joint result of the actions of all the undertakings, i.e., those by consumers, ISP, and content providers. The role of consumers in limiting the congestion of the network has been developed by [Jullien and Sand-Zantman](#page-31-5) [\(2018\)](#page-31-5). Here, we focus on the supply side, by looking at the extent to which various regulations can give CPs more or fewer incentives to exert some congestion-reducing effort. In this vein, [Jullien and](#page-31-6) [Bouvard](#page-31-6) [\(2022\)](#page-31-6) examines a network cost-sharing mechanism between a content provider and a network operator, whose main property is to strengthen the content provider's incentives to moderate traffic. The general idea that CPs could influence the congestion of the network was discussed by [Peitz and Schuett](#page-32-3) [\(2016\)](#page-32-3). In their article, this action was designed to guarantee that the content was delivered in case of congestion, and the lack of coordination was at the source of what the authors called the "inflation of traffic". In our approach, the effort of content providers reduces congestion instead of increases it. We show that, as in [Choi, Jeon, and](#page-31-7) [Kim](#page-31-7) [\(2018\)](#page-31-7), there can be some complementarity or substitutability between the effort exerted by the content providers to reduce the congestion on the network and the ISP's investment decision. This interplay between the ISP and the content providers' action is analyzed in a setting of heterogeneity across content providers. A last important aspect of our work is related to the environmental impact of the industry; for this, we assume that the environmental damage depends on the size of the network. This is an important element of our study since, in contrast to the standard net neutrality debate, increasing the size of the network may not be the goal of a regulatory change.

This paper is also related to the growing debate on the impact on digitization^{[1](#page-5-0)} on carbon footprint and the electricity consumption. [Ahmadova et al.](#page-30-2) [\(2022\)](#page-30-2) recent contribution analyze mostly empirically the impact of digitalization on environmental performance at a country-level. They show that the effect has a inverted U shape with two effect. First, the impact is positive as digitalization leads to better management of resources. Then, the second effect is negative because the resource usage increases. Our results are in line with the idea that the impact of digitization is non linear and can be positive or negative. A major difference is that we focus on a theoretical approach and consider different institutional environments at the sector level. [Lange, Pohl, and Santarius](#page-31-8) [\(2020\)](#page-31-8) focus more specifically on the link between digitalization and energy consumption. They identify theoretically four effect of digitalization, some of them leading to increase energy consumption, and others point on the opposite direction. We share with this work the idea that digitalization may have an ambiguous effect and also develop a theoretical model to analyze this. But we insist on the way the economic organization and regulation of Internet market can affect the environmental performance. [Santarius, Pohl, and](#page-32-4)

 $¹$ [Another debate present in the literature is the impact of digitization and environmental regulation](#page-32-4)</sup> [on the productivity of firms. See Wen, Wen, and Lee \(2022\)](#page-32-4)

[Lange](#page-32-4) [\(2020\)](#page-32-4) study to which extend digitalization could foster or alter the prospect of decoupling and how to make ICT an asset to make the economic system more sustainable. We also discuss how higher internet usage could be compatible with lower environmental impact but from a market and regulatory view point. We focus on the role of incentives at the firm level, and provide conditions under which higher ICT usage could be compatible with lower environmental damages.

In Section [2](#page-6-0) we describe our model, whereas Section [3](#page-10-0) presents two main benchmarks; the optimal allocation and the outcome under net neutrality. In Section [4,](#page-13-0) we discuss the laissez-faire approach and compare it to the previous cases focusing on the incentives CPs have to make congestion-reducing efforts and, as a consequence, on the equilibrium environmental footprint. In Section [5,](#page-19-0) we study price-based regulation and show when it contributes to improving the allocation compared to the net neutrality case. Section [6](#page-26-0) examines the important issue of potential exclusion of CP's that may arises in each regimes studied so far. Section [7](#page-29-0) concludes. All proofs are relegated to the Appendix.

2 Model

Internet Service Provider (ISP) We consider a model in which a monopoly ISP contracts with consumers and allows some content providers to access its consumers. The ISP owns a network for which it must choose the total capacity *K*. This capacity can be built or extended at an increasing and nonconcave cost *C*(*K*) and, without loss of generality, we assume that $C(K) = cK$. The investment process in broadband capacities generates negative environmental impacts. More precisely, we assume that a network of capacity *K* generates some damages of CO[2](#page-6-1)-GHG emissions $D(K) = \delta K^2$ Here δ is a factor that measures the environmental impact of investments in broadband. This factor is meant to represent not only the cost generated to the network expansion but also the negative externality induced by the extra consumption in energy as well as by the additional devices construction (screens, phones, computers)

²Using a more general damage function, increasing and convex, would not qualitatively alter our main results, but at the cost of an unnecessary increase in the complexity of the analysis mainly when defining the first-best outcome.

this expansion triggers. As this factor can hardly be controlled by the ISP, we will take the environmental impact δ per unit of capacity as given.

Content providers (CPs) The capacity chosen by the ISP is used by many CPs to reach consumers. But different CPs have different effects on the network, some of them needing more capacity than others. More precisely, the link between the amount of content consumed (and therefore the satisfaction consumers can derive) and the impact on the network is not uniform across CPs. We model this by assuming that when a quantity *q* of content is consumed, the ISP capacity usage is θ*q*. Here, the parameter θ represents the gross congestion impact of consumption and varies across CPs. To keep the analysis simple, we assume that there are two types of CPs, that is $\theta \in \{\theta, \overline{\theta}\}\$, where $\overline{\theta} > \theta$ and there is a share of $\mu \in (0, 1)$ of CPs with a factor θ and $(1-\mu)$ with a factor $\overline{\theta}$. Moreover, we denote by $\mathbb{E}(\theta)$ the average value of θ .

An important feature of our model lies in the fact that the CPs can affect the congestion impact. Indeed, the gross congestion impact can be reduced by some data compression techniques, modeled by an effort $e \in \{0, \hat{e}\}\)$ chosen by the CPs.^{[3](#page-7-0)} Therefore, the net congestion per unit of consumption is given by $z(\theta) = (\theta - e)$ for a type- θ usage. Reducing the congestion is costly and this unit cost is type-dependent, $\psi(\theta)$. To represent the fact that load reduction is easier for capacity-intensive CPs, we assume that $\psi(\overline{\theta}) < \psi(\underline{\theta})$. We denote $\psi(\overline{\theta}) = \overline{\psi}$ and $\psi(\underline{\theta}) = \psi$, so our assumption boils down to $\psi > \overline{\psi}$. Therefore, the ISP faces a capacity constraint that writes as

$$
\mu z\left(\underline{\theta}\right)q\left(\underline{\theta}\right)+(1-\mu)z\left(\overline{\theta}\right)q\left(\overline{\theta}\right)\leq K
$$

where $q(\theta)$ denotes the consumption of θ content.

Finally, the CP business model is based on online ads with a monetary value of $b(\theta)$ per unit of consumption *q*. For most of the paper, we will assume that *b* is the same across CPs, as we want to emphasize CP heterogeneity in another dimension. Still, we will discuss how some of our future results could be affected when the revenues per unit differ across CPs.

 3 This simple model with only two possible levels of effort and linear costs can be generalized to a continuous structure for efforts i.e. $e \in [0,\hat{e}]$ and cost functions $\psi(\theta)s(e)$ increasing and convex in e without altering our results.

We make two assumptions on the ad revenues of the CPs. First, we assume that $c\theta - b(\theta) > 0$ for all c, θ, b . This means that, if consumers do not derive any utility from consumption, it is not optimal to built any capacity regardless of the congestion generated by the CPs. This assumption is necessary in our setting to avoid an infinite choice of capacity even when consumers do not care about content. Second, we assume that $b(\theta) \geq \min{\lbrace \theta \psi, \bar{\theta} \bar{\psi} \rbrace}$, i.e., the revenues generated by the ads are not too small. This does not mean that all CPs can finance the cost of reducing congestion but that all CPs could finance the lowest cost of congestion. This assumption is not crucial to derive most of our results but limits the number of cases to study.

Consumers We assume that consumers subscribe to the network at a fixed price *T* and derive a gross utility *u*(*q*(θ)) when they consume a quantity *q*(θ) of a content proposed by a type- θ content provider. We assume that $u'(q) > 0, \forall q \ge 0$ and that $u''(q) < 0$. In our analysis, the degree of concavity of the utility function will play an important role. A higher degree of concavity in the neighbourhood of the equilibrium quantity will mean that the service provided is close to saturating needs, whereas a lower degree of concavity leaves more room for a socially desirable increase in the quantity of service provided.

When choosing how much to consume, it is reasonable to consider that consumers do not take into account the impact of each type of CP on network capacity. Therefore, we model their choice as a simple problem of determining consumption levels of each content subject to a content capacity *k* supplied by the ISP for their usage. An important point is that *k* differs from *K*. Indeed, *k* represents the amount of content that consumers can consume when the network capacity is *K*. To go from *k* to *K*, one should weight each content by its congestion coefficient. Note also we assume that consumers are impacted by the negative environmental externality caused by the network capacity, but this does not affect their consumption levels. [4](#page-8-0) When consumers can freely choose their usage levels, their consumptions, denoted

⁴ If the externalities were related mainly to the consumption (*k*), they would have an impact on the tariff proposed by the ISP, and therefore on its choice of network size.

 $q(\theta) = q$ and $q(\bar{\theta}) = \bar{q}$ for each type of CP, are such that

$$
\max_{\underline{q},\overline{q}} U = \mu u\left(\underline{q}\right) + (1-\mu) u\left(\overline{q}\right) - T - \delta K
$$

subject to

$$
\mu q + (1 - \mu) \overline{q} \le k
$$

Direct computations lead to $q = \overline{q} = k$.

In this case, the ISP will face the following constraint:

$$
\left[\mu\left(\underline{\theta} - e(\underline{\theta})\right) + (1 - \mu)\left(\overline{\theta} - e(\overline{\theta})\right)\right]k \le K\tag{1}
$$

where $e(\theta)$ and $e(\bar{\theta})$ correspond to the congestion-reducing effort of each type of CP. In this article, we do not investigate in depth how the ISP sets its price for subscribers or how different incentive schemes could make consumers more responsive to the congestion issues (see [Jullien and Sand-Zantman, 2018,](#page-31-5) on this point). Therefore, to focus on the relationship between the CPs and the ISP, we assume that the ISP can extract the whole surplus from consumers by setting a tariff $T = u(k)$.^{[5](#page-9-0)}

Timing of players' move Except for the first-best described in the next section, we consider the following time of events

- 1. The ISP invests in network capacity *K*.
- 2. The content providers chose their congestion-reducing effort
- 3. The ISP proposes a contract (*T*, *k*) to consumers where *T* is the price and *k* the maximum amount of content consumed.
- 4. Consumers accept of refuse the contract, choose their consumption for each content providers and pay the ISP the price if they accept.

⁵Note that the gross surplus of consumers is defined as the difference between the net utility when they opt in and consume, i.e. $u_b = u(k) - \delta K$ and the one outside of the contract, the externality being borne, i.e. $u_0 = 0 - \delta K$. So the gross surplus from consuming writes $v = u_b - u_0 = u(k)$. We could also assume that the ISP captures only a share of this surplus, without changing the main message of this article.

In the next sections, we will analyse how allowing the ISP to price for congestion or the regulation to intervene more directly before the CPs make their choice will affect the outcome of this game.

3 Benchmarks

In this section, we derive two useful benchmarks. First, we compute the allocation that maximizes social welfare, and then we derive what happens under net neutrality.

3.1 Optimal Allocation

As a starting point, it is important to derive what the optimal allocation could be. In our model, this means 1) whether the CPs should invest in order to reduce the capacity on the network, 2) what size of network should be chosen, and 3) how much time consumers should spend on each type of CP. Formally, we define the first-best allocation as the actions $\{q(\theta), e(\theta), K\}$ that maximize social welfare subject to the ISP capacity constraint

$$
W = \mu \left[u \left(q(\underline{\theta}) \right) + \left(b(\underline{\theta}) - \underline{\psi} e(\underline{\theta}) \right) q(\underline{\theta}) \right] + (1 - \mu) \left[u \left(q(\overline{\theta}) \right) + \left(b(\overline{\theta}) - \overline{\psi} e(\overline{\theta}) \right) q(\overline{\theta}) \right] - (\delta + c) K
$$

subject to

$$
\mu\left(\underline{\theta} - e(\underline{\theta})\right)q(\underline{\theta}) + (1 - \mu)\left(\overline{\theta} - e(\overline{\theta})\right)q(\overline{\theta}) \le K
$$
\n(2)

In the above objective, all the direct and indirect consequences of investment, effort and consumption are taken into account. Note also that the constraint clearly shows an interplay between congestion and network size, an important aspect we will comment on later. Note also that the social optimum should account for the environmental impact of the network size (or expansion).

Solving this problem leads to this first proposition.

Proposition 1. Let us define for all θ $q_0(\theta)$ and $q_1(\theta)$ by

$$
u'(q_0(\theta)) = (\delta + c)\theta - b(\theta)
$$

$$
u'(q_1(\theta)) = (\delta + c)\theta + (\psi(\theta) - (\delta + c))\hat{e} - b(\theta)
$$

Then, the optimal allocation is such that

• If
$$
\underline{\psi} > \overline{\psi} > \delta + c
$$
, then
 $e(\theta) = 0$ for all θ , $q(\theta) = q_0(\theta)$, and $K_0^* = \mu \underline{\theta} q_0(\underline{\theta}) + (1 - \mu) \overline{\theta} q_0(\overline{\theta})$.

\n- \n
$$
If \psi > \delta + c > \overline{\psi}, then
$$
\n
\n- \n $e(\theta) = 0 \text{ and } e(\overline{\theta}) = \hat{e}, q(\theta) = q_0(\theta), q(\overline{\theta}) = q_1(\overline{\theta}), \text{ and } K_{01}^* = \mu \theta q_0(\theta) + (1 - \mu) \left(\overline{\theta} - \hat{e} \right) q_1(\overline{\theta}).$ \n
\n- \n $If \delta + c > \psi > \overline{\psi}, then$ \n
\n- \n $e(\theta) = \hat{e}, q(\theta) = q_1(\theta), \text{ and } K_1^* = \mu \left(\underline{\theta} - \hat{e} \right) q_1(\underline{\theta}) + (1 - \mu) \left(\overline{\theta} - \hat{e} \right) q_1(\overline{\theta}).$ \n
\n

The optimal allocation internalizes all the externalities that any action can generate. First, the optimal efforts by the CPs reflect their impacts not only on the ISP cost but also on the environment. For each CP, the first-best level of effort depends on a trade-off between the private cost of effort and the impact on the network costs, both the construction and the environmental costs. When these impacts are low, the optimal solution entails zero efforts for both CPs. As the impacts increase, it is optimal to request an effort from the high-type CPs and then, when the impacts are very strong, an effort from the low-type CPS. Second, the usage levels depend on their effects on all the agents. Indeed, this level increases with the consumers' marginal utility and the ad revenues generated by the CPs, whereas it decreases with the cost of building the capacity and the environmental damage.

In what follows, we will sometimes use a more specific objective function representing a environmentally-driven planner. To that end, we define a consumeristenvironmentalist regulator as an agent only taking into account the consumer surplus and both the building and environmental costs of the capacity. With such a planner, the welfare function is given by:

$$
W_e = \mu u(q(\underline{\theta})) + (1 - \mu) u(q(\bar{\theta})) - (\delta + c)K
$$
\n(3)

Note that the first-best outcome that results from this criterion is simply described by Proposition [1](#page-10-1) taking $b(\theta) = 0$ so that optimal efforts are always maximal.

In the rest of this article, we study to what extent a monopoly ISP, whether unregulated or regulated, has incentives to make the same (or relatively similar) choices as a planner.

3.2 Net Neutrality

As a second benchmark, we look at a decentralized situation in which not only there is no central planner to control the actions of the agents but also the ISP is forbidden to charge any fee to the CPs. This situation, called net neutrality, can be considered to be the current situation in Europe and in the U.S. Indeed, even if CPs play a key role in bringing some content to consumers and therefore use the network more than the standard internet user, ISPs are not allowed to charge them termination fees for their connection to the ISP's subscribers. What are the resulting allocations and inefficiencies in this case?

In this setting, the revenue the ISP earns come only from its subscribers. As explained in Section 2, the ISP can extract the whole consumer surplus. Since consumers do not internalize the impact of their consumption on the network, they consume the same quantity k for any type of \mathbb{CP}^6 \mathbb{CP}^6 . This implies that the ISP will establish a subscription fee equal to *u*(*k*) and its profit are simply given by

$$
\Pi = u(k) - cK.
$$

The only choice the ISP has is to set the size of the network, which allows consumers to obtain a level of usage *k* and therefore a corresponding level of gross surplus *u*(*k*).[7](#page-12-1) The link between *k* and *K* depends on the congestion factor that is not controlled by the ISP. Indeed, this congestion is a function of the type of content available and consumed, and of the effort exerted by the CPs to reduce the load they generate. As reducing the load is costly, and the CPs have no personal or financial incentives to bear this cost, there will be no effort on their side. This implies that the ISP will maximize its profit under the constraint

$$
k\mathbb{E}\left(\theta\right) \leq K
$$

This problem is easily solved and leads to the following proposition.

 6 In fact, consumption could differ from one type of CP to another according to the user's taste. It is only to simplify the analysis, and without loss of generality, that we assume consumers have the same satisfaction from every type of content.

 7 Therefore when we compare consumption levels between different regulatory regimes, this will give us a direct comparison of gross consumer surpluses.

Proposition 2. *In a net neutrality regime, the equilibrium consumption and investment levels are given by*

$$
u'(k^n) = \mathbb{E}(\theta) c; q^n(\theta) = k^n \text{ and } K^n = \mathbb{E}(\theta) k^n
$$

In this net neutrality situation, neither the consumption level (in general), nor the choice of effort by the CPs nor the choice of investment by the ISP are optimal. Indeed, the CPs do not exert any congestion-reducing effort. This may be optimal, say when the cost of both types of CP are high, but this is not the case in general. Second, consumers do not adjust their consumption to the load, as there are no reasons to do so. Their consumption patterns only depend on their taste, not on the different cost/load they could generate on the network by consuming, for example, more capacity-intensive content. Finally, the ISP, when choosing its network size, only takes into account the congestion factor for each type of content, but neither the benefit that accrues to CPs when there is more consumption (the *b*) nor the marginal environmental cost of the capacity (δ) . One cannot tell, in general, whether this leads to an upward or a downward distortion of the network size. But the situation is very far from optimal, as none of the many externalities (on the load, or on the environment) are considered.

In the next sections, [4](#page-13-0) and [5,](#page-19-0) we focus on situations such that the CPs do not differ in the revenue they generate from their business model $(b(\theta) = b(\theta) = b)$. This assumption allows us to focus on the main trade-offs and postpone the discussion about potential exclusion of some CPs, exposed in Sections [6.](#page-26-0)

4 Laissez-faire

In this section, we investigate two laissez-faire options: the first corresponds to a situation in which the ISP can charge the CPs uniform prices regardless of the congestion CPs generate on the network, and in the second, we allow the ISP to fine tune its pricing to take into account CP heterogeneity.

4.1 Uniform prices

We assume that a *uniform price p* can be set by the ISP to allow CPs be connected to their network. This uniformity assumption can be justified when the ISP cannot perfectly observe (or contract on) the level of congestion the CPs generate or simply because the ISPs might not be allowed to price discriminate among the CPs. In this setting, the ISP profit writes as

$$
\Pi = u(k) + pk - cK.
$$

To maximize this profit, the ISP then chooses the price and the size of the network, taking into account the congestion constraint [\(2\)](#page-10-2) and the CPs profitability constraint

$$
\pi(\theta) = (b - p - \psi(\theta)e(\theta))k \ge 0
$$
\n(4)

As in the case of net neutrality, the CPs have no incentives to exert any congestionreducing effort, so $e(\theta) = 0$. The equilibrium outcome in this case is described below.

Proposition 3. *When the ISP can charge the CPs, the consumption and equilibrium investment are given by*

$$
q^{u}(\theta) = k^{u} : u'(k^{u}) + b = \mathbb{E}(\theta) c
$$

and $K^{u} = \mathbb{E}(\theta) k^{u}$

As a result, $K^u > K^n$ *and* $k^u > k^n$ *.*

When the ISP can charge the CPs, it takes into account the revenues generated from ads when choosing the size of the network. As more consumption generates more ad revenues, a higher network size allows increased consumption, and the ISP chooses to invest more than in the net neutrality regime. When the ISP can charge the CPs, it internalizes the money CPs can generate. But by comparison with the first-best allocation, there are still two unsolved issues. First, the CPs are given no incentive to exert any congestion-reducing effort. Second, the negative environmental impact is not internalized by the ISP. Therefore, focusing on the environmental side, this laissez-faire situation creates more damages than the net neutrality situation.

4.2 Tailored prices

We now assume that the ISP can use the ex post CPs' impact $z = \theta - e(\theta)$ to adjust prices. Thus, the ISP can propose type-specific two-part tariffs $T(z) = p(\theta)z + t(\theta)$, with $p(\theta) \ge 0$ and $t(\theta) \ge 0$. This new pricing system creates some incentives for the CPs to exert their congestion-reducing effort. Indeed, the CPs profit is given by

$$
\pi(\theta) = (b - p(\theta)(\theta - e) - t(\theta) - \psi(\theta)e)k.
$$

It is direct to see that $\frac{\partial \pi(\theta)}{\partial e} = p(\theta) - \psi(\theta)$. Hence if $p(\theta) \ge \psi(\theta)$ then $e(\theta) = \hat{e}$ and if $p(\theta) < \psi(\theta)$ then $e(\theta) = 0$. Moreover, with the fee $t(\theta)$, the ISP is then able to capture the remaining profit of the CPs by setting $t(\theta) = b - p(\theta)(\theta - e) - \psi(\theta)e$. We now discuss in detail what choice the ISP will make in this context.

It is important to see that the ISP may not be interested in inducing the CPs to exert congestion-reducing effort. Indeed, even if this effort reduces the need for network expansion, it deprives the ISP from some revenue. We can therefore state the following lemma.

Lemma 1. *When the ISP can set some congestion-based prices, a type-*θ *CP will be incentivized to exert some congestion-reducing effort if and only if* $c \geq \psi(\theta)$ *.*

Note that we have assumed that the ad revenues are not too small, or more precisely, that $b \ge \min{\{\theta\psi, \theta\overline{\psi}\}}$. This implies that the ISP will be able to induce some effort of at least one type of CP (otherwise, we would return to the uniform-price case studied above).

Suppose, first, that for all CPs, the ad revenues are large enough to finance the cost of the congestion-reducing effort, i.e., if $b > \max{\lbrace \theta \psi, \theta \overline{\psi} \rbrace}$. Then the ISP always wants to induce this effort, as by assumption $c\theta > b$, which leads to $c > \psi$. Consequently the ISP profit writes as

$$
\Pi = u(k) + \mathbb{E}(b - \theta \psi(\theta))k - cK \text{ s.t. } k(\mathbb{E}(\theta) - \hat{e}) \leq K
$$

Proposition 4. *Suppose that* $b > \max{\lbrace \underline{\theta}\psi, \overline{\theta\psi} \rbrace}$ *and that the ISP can use congestion-based prices. Then, if c* > ψ*, all the CPs are induced to exert some congestion-reducing e*ff*ort, the*

ISP will capture all the CPs' profits and the equilibrium consumption and investment levels are given by

$$
kt : u'(kt) = (\mathbb{E}(\theta) - \hat{e})c - \mathbb{E}(b - \theta \psi(\theta))
$$

$$
Kt = kt (\mathbb{E}(\theta) - \hat{e}).
$$

 ω here k^t > kⁿ. Moreover,K^t ≤ Kⁿ if and only if u is concave enough and 0 < $\mathbb{E}\left(b-\theta \psi(\theta)\right)$ ≤ \overline{B} *, where* $\overline{B} = (\mathbb{E}(\theta) - \hat{e})c - u'\left(k^n \frac{\mathbb{E}(\theta)}{\mathbb{E}(\theta)}\right)$ $\frac{\mathbb{E}(\theta)}{\mathbb{E}(\theta)-\hat{e}}\Big).$

Here, the ISP can capture all the rents after congestion reduction due to the CPs' effort. Consequently, it may prefer to induce these efforts and capture the remaining rents instead of investing in costly capacities. If expected ex post rents are not too high ($\mathbb{E}\left(b - \theta \psi(\theta)\right) < \overline{B}$), this has a dampening effect on the network size. Moreover, this is not done at the expense of consumption, which increases thanks to the reduction in congestion. As a result, tailored price discrimination can be environmentally friendly. However, if the expected ex post rents are high or if consumer utility is not concave enough, the ISP has more incentives to increase the consumer's demand, boosting capacity investments above the net neutrality level.

Let us suppose now that if the ISP cannot induce all CPs to exert some effort, i.e., that there exist $\tilde{\theta}$ such that $b < \tilde{\theta}\psi(\tilde{\theta})$. Then, for this type of CP, the ISP will choose not to induce any effort by setting $p(\tilde{\theta}) = 0$ for this type of CP capture all its profit through the fee $t(\tilde{\theta}) = b$. Now the ISP profit writes as

$$
\Pi = u(k) + bk - \theta \psi(\theta) k - cK \text{ s.t. } k(\mathbb{E}(\theta) - m(\theta)\hat{e}) \le K
$$

with $\theta \neq \tilde{\theta}$ and $m(\theta)$ is the share of CPs that exert the congestion-reducing effort.

 $\bf{Corollary~1.~}$ *Suppose that there exists one type* $\tilde{\theta}$ *such that b* − $\tilde{\theta}\psi\left(\tilde{\theta}\right)$ < 0 *and that the ISP can use congestion-based prices. Then only those CPs with* $\theta \neq \tilde{\theta}$ *are induced to exert some congestion-reducing e*ff*ort, but the ISP will still capture all the CPs' profit. The equilibrium consumption and investment levels are given by*

$$
k^{\tau} : u'(k^{\tau}) = c \left(\mathbb{E}(\theta) - m(\theta) \right) \hat{e} - \left[b - \theta \psi(\theta) \right]
$$

$$
K^{\tau} = \left(\mathbb{E}(\theta) - m(\theta) \hat{e} \right) k^{\tau}
$$

with $k^{\tau} > k^n$. Moreover, if *u* is concave enough and $b - \theta \psi(\theta) \leq \underline{B}$ then $K^u > K^n \geq K^{\tau}$ *where* $\underline{B} = (\mathbb{E}(\theta) - \hat{e})c - u'\left(k^n \frac{\mathbb{E}(\theta)}{\mathbb{E}(\theta) - m}\right)$ $\frac{\mathbb{E}(\theta)}{\mathbb{E}(\theta)-m(\theta)\hat{e}}$.

When some CPs cannot afford to exert congestion-reducing effort, the ISP still captures their profit and induces the other CPs to exert some effort. This situation can still induce less investment in capacity while increasing gross consumer surplus, but the conditions to obtain this results are more stringent than in Proposition [4.](#page-15-0) Note that the same result is obtained if $c \in [\overline{\psi}, \psi]$. In this case, only the CPs with type θ will be incentivized to exert congestion-reducing effort. At last, for *c* < ψ, the ISP will set $p = 0$ for all types and uses only uniform prices as in the previous section.

What we can say about the impact of this laissez-faire regulation on the environment? First, the ISP should be given enough flexibility to be able to induce CPs to exert some congestion-reducing effort. In this respect, tailored prices are more efficient than uniform prices. Second, the revenues generated by the CPs (or at least the share the ISP can capture) should not be too high. Otherwise, the ISP will be tempted to focus on revenue maximization instead of incentivizing the CPs to reduce the externalities they generate on the network and, therefore, on the environment. When these two conditions are satisfied and consumer utility is concave enough, laissez-faire generates less environmental damage than under net neutrality.

4.3 Complementary or substitutability between eff**ort and network size**

It is important to come back to interplay between the effort of the CPs and the optimal consumption level. Indeed, when the CPs exert an effort to reduce congestion, it also reduces the cost of building some extra capacity, thereby increasing the usage levels. This result is important as it is at the core of the preceding results, in particular for Proposition [4.](#page-15-0) Therefore, it is interesting to examine when the effort exerted by the CP will increase or decrease the equilibrium level of capacity.

Corollary 2. *If u is concave enough, i.e., the marginal benefit of consumers is decreasing sharply, the equilibrium size of the network decreases with the congestion-reducing e*ff*ort of the CPs.*

The effort exerted by the CPs has two effects on the final allocation. First, for a given level of consumption, it decreases the incentives to build a large network and

therefore leads the ISP to choose a lower *K*. We can refer to this as the *congestion-based e*ff*ect*. Second, this effort decreases the perceived cost of increasing the network size, and therefore the marginal cost of consumption; so, the optimal consumption levels increase, and we can refer to this as the *consumption-based e*ff*ect*. When the marginal utility is strongly decreasing (*u* concave enough), the optimal consumption level does not vary much when the building cost increases, so this latter effect is offset by the former and induces more effort from the CPs to lead the ISP to decrease the network size. This first case is illustrated in Figure [1,](#page-19-1) with $\eta = \hat{e}c + \mathbb{E}(b - \psi\hat{e})$. When the marginal utility is weakly decreasing, the reverse holds and a reduction in the level of congestion is associated with an increase in the optimal size of the network. This second case is illustrated in Figure [2.](#page-20-0) This means that efforts to reduce congestion are substitute (resp. complement) for investment in the network when consumers are only slightly (resp. very) impacted by small changes in the quantities consumed.

Figure 1: Substitutability between congestion-reducing effort and network size

5 Environmental-based regulated prices

In the previous section, we showed that laissez-faire could lead to different outcomes, some of them quite satisfactory compared to net neutrality, and others less satisfactory. One key element is that laissez-faire does not guarantee that the ISP

Figure 2: Complementarity between congestion-reducing effort and network size

will induce the CPs to exert some congestion-reducing effort. In so far as this is a necessary condition to reconcile the environmental requirement and the consumers' needs, we consider an alternative option based on regulatory prices. In the next subsection, we first determine the set of prices that a regulator might consider to foster congestion reducing effort, and then the ones that a consumerist-environmentalist (hereafter C-E) regulator would pick. Then, we provide a general comparison of our main outcomes with respect to the surplus derived by consumers, the C-E welfare and the standard social welfare criteria.

5.1 Feasible and optimal regulated prices

More precisely, we assume that the regulator relaxes the internet rules, allowing that a price be charged to the CPs. In contrast to the previous section, however, this price must be based only on the ex post congestion impact $z = \theta - e$, and set by the regulator. The amount paid by a CP is then written as $t(z) = pz$, where p is the same across all CPs, and the CP profitability constraint is given by

$$
\pi(\theta) = (b - p(\theta - e) - \psi(\theta)e) k \ge 0.
$$

This price regulation creates some incentives for the CPs to exert some congestionreducing effort. As before, for a price per unit of congestion *p*, we have $\frac{\partial \pi(\theta)}{\partial e} = p - \psi$. Hence, if $p \ge \psi(\theta)$ then $e(\theta) = \hat{e}$.

We assume that the money collected is not captured by the ISP. Moreover, as we do not want to introduce additional motives for taxation, we do not consider any cost of public funds (see [Browning, 1976\)](#page-31-9). Note that the financial burden imposed on the CPs could lead some of them to exit the market (we referred to this as exclusion in the previous section). This would be the case if their revenues (*b*) were small enough. We focus here on the situation in which exclusion is not an issue – the most interesting and relevant case – and consider the alternative case in the next section.

Regulation aims at choosing a price to increase efforts from CPs. Therefore, we will study two possible regulated prices given by $p^r = \psi$ and $p^r = \overline{\psi}$. If $p^r = \psi$, all CPs exert the effort \hat{e} and the ISP phroblem writes as

$$
\max_{k,K} \Pi = u(k) - cK \text{ s.t. } k(\mathbb{E}(\theta) - \hat{e}) \le K
$$

This price is feasible if capacity-intensive CPs remain profitable. A condition for this to be true is $b \ge \psi \overline{\theta} + (\psi - \overline{\psi})\hat{e} > \psi \overline{\theta}$.

Now, if $p^r = \overline{\psi}$, the capacity-economical CPs (θ) do not exert the effort but are

still profitable. The ISP profit is unchanged, but its constraint is now written as

$$
k(E(\theta) - (1 - \mu)\hat{e}) \le K
$$
 (5)

The next proposition describes the consequence of setting these two prices.

Proposition 5. *When the regulator uses congestion prices, then (a) for* $p^r = \psi$ *, the consumption and equilibrium investment are given by*

$$
u'\left(\underline{k}^r\right) = \left(\mathbb{E}\left(\theta\right) - \hat{e}\right)c \text{ and } \underline{K}^r = \underline{k}^r \left(\mathbb{E}\left(\theta\right) - \hat{e}\right)
$$

(b) for $p^r = \overline{\psi}$, the consumption and equilibrium investment are given by

$$
u'\left(\overline{k}'\right) = \left(\mathbb{E}\left(\theta\right) - (1 - \mu)\hat{e}\right)c \text{ and } \overline{K}' = \overline{k}'\left(\mathbb{E}\left(\theta\right) - (1 - \mu)\hat{e}\right)
$$

(c) If u is concave enough, then $\underline{k}^r > \overline{k}^r > k^n$, $\underline{K}^r < \overline{K}^r < K^n$, and $\underline{\Pi} > \overline{\Pi} > \Pi^n$.

Price regulation is intended to create direct incentives for CPs to exert some effort. If the regulated price is based on the highest cost ψ , all CPs reduce their impacts on the network (as long as they remain profitable), and when the consumer marginal benefit decreases sharply, the ISP reduces its investment level as the congestionbased effect dominates the consumption-based effect (see the discussion at the end of Section 4). The ISP constraint is relaxed, which, compared to net neutrality, allows for savings in capacity investments and an increase in consumer traffic. If the regulated price is based on the lowest cost $\overline{\psi}$, no congestion-reduction incentives are given to the capacity-economical CPs. Therefore, the congestion-based effect still applies but is weakened.

In this context, the socially optimal choice depends on the society's preferences. We focus here on the case of a C-E regulator defined in [\(3\)](#page-11-0) who only takes into account the standard consumer surplus and both the building and environmental costs of the capacity. Applying directly the results in Proposition [5,](#page-22-0) one can state the following.[8](#page-22-1)

⁸The result of the following corollary extends to any welfare function γ(*u*(*k*) − *cK*) − (1 − γ)δ*K* for any $\gamma \in [0, 1]$, where γ would be the degree of consumerism of the regulator.

Corollary 3. *If the revenues generated by the CPs are high enough to avoid exclusion, i.e.,* if $b \ge \psi \overline{\theta} + \left(\psi - \overline{\psi}\right)$ ê, a consumerist-environmentalist regulator chooses $p^r = \psi.$

A C-E regulator has social preferences oriented toward a dual objective: to increase consumer traffic and to decrease the network size. As long as all CPs remain profitable when the congestion regulation is implemented, this regulator prefers to make the congestion-based effect fully effective, as it is strongly aligned with its preferences. To that end, choosing $p^r = \psi$ is the optimal policy and clearly improves upon the current net neutrality situation.

To which extent this environmental-based regulation could be implemented? Even if it appears to be the most effective, it does not derogate from the general problematics of regulation, particularly incentive-based regulation. On the one hand, the regulator could have information gaps on congestion conditions, which, as we know, would lead to transitory inefficiencies of this policy. The regulator would be forced either to delegate its implementation to ISPs, and thus reward them for it, or to give up informational rents to CPs. On the other hand, it might not have the legal tools to impose prices as we have assumed here. For CPs headquartered outside its local jurisdiction, it will be difficult for a local regulator to enforce environmentalbased pricing. Here again, it may be forced to delegate implementation to the ISP.

5.2 Comparaison of outcomes

We now propose to compare our main outcomes with regards to the consumer' surplus, the consumerist-environmentalist welfare and the standard social welfare. More precisely we compare all regimes: Net Neutrality (Proposition 2) , Uniform Prices (Proposition 3), Tailored Prices (Proposition 4), Regulated Prices (Corollary 3 of Proposition 5).

Consumer surplus

Indeed, consumer surplus can be seen either gross (which is directly related to consumption), or net which is the opposite of capacity. So in this context, the latter is just the environmental criterion. For the gross consumer' surplus criterion, as noted

in footnote [7,](#page-12-1) comparing consumption levels is sufficient. Then one can state the following result.

Corollary 4. *Comparison of consumer's surplus depends on the costs and e*ff*ort parameters. Net neutrality always yields to the lowest level of surplus for consumers.*

- 1. If $\mathbb{E}(\theta \psi(\theta)) \geq \hat{e}c$, the laissez-faire with uniform prices regime yields the highest level *of surplus for consumers*
- 2. If $\hat{e}c > \mathbb{E}(\theta \psi(\theta))$, the laissez-faire with tailored prices regime yields the highest level *of surplus for consumers*

As far as consumption is concerned, a laissez-faire policy, either with uniform or tailored prices, is preferred to regulation or net neutrality. This is because with laissez-faire, the ISP can recover their rents, which are proportional to the level of supply. This has a positive impact on the amount of services proposed by the ISP in the contract to consumers. Neither price regulation nor net neutrality allow the ISP to capture these rents, therefore these regimes are dominated from the consumer surplus criterion. A second result of the Corollary is that laissez-faire with uniform prices is preferred when network building costs are low, and laissez-faire with tailored prices regime when there are high costs. The intuition is that when network building costs are high, the size of the network is limited, there is a need to foster CPs congestion-reducing effort to achieve high level of consumption.

Consumerist-environmentalist welfare

As defined in [\(3\)](#page-11-0), the consumerist-environmentalist welfare W_e balances consumption and environmental damagse due to the building of network capacity. Comparing all C-E welfare levels for our main outcomes allows to state the following result.

Corollary 5. *Comparison of C-E welfare depends on the desirability of services by consumers.*

1. If u is concave enough, environmental-based price regulation yields the highest level of C-E welfare.

2. Otherwise, net neutrality yields the highest level of C-E welfare.

A C-E regulator that seeks a balance between consumption and environmental damage, will prefer to implement regulated prices if the utility is highly concave, that is when provided services are close to saturating needs. Indeed, in this case, price regulation allows the congestion-based effect of CPs efforts to dominate the consumption-based effect. As a result, repealling net neutrality in favor of a price regulation regime leads to a mild increase in consumption but a significant reduction in environmental damages related to the capacity. In the contrary, if the utility is slightly concave, that is when consumers have a high desirability of services provided, the lowest installed capacity is achieved with net neutrality. The consumption-based effect dominates and any mitigation effort realized by the CPs cannot curb the increase in consumption that follows, implying an increase in the environmental damages due to the capacity. Consequently, repealling net neutrality would be inefficient from a C-E regulator point of view.

Social welfare

To provide a final comparison of the various studied alternatives, we consider here the standard social welfare W^9 W^9 which on top on the previous criteria incorporates the profits of all companies in the sector. Without stating a formal result, we assert that this comparison lies between that previously established according to consumer surplus and that established according to the C-E welfare criterion. Indeed, for a given level of consumption *k* and when the CPs exert a effort $e(\theta)$, the standard social welfare can be written as $W = W_e + \mathbb{E} (b - \psi(\theta) e(\theta))k$. As a result, when the the CPs business revenues *b* are very high, i.e. CPs activities are very profitable, the social welfare is colinear to the consumption level *k*. Therefore the results in Corollary [4](#page-24-0) also apply in terms of social welfare. On the contrary, if *b* is small, i.e. close to min{ $\theta \psi$, $\theta \psi$ }, then the social welfare is merely equal to the C-E welfare, the results in Corollary [5](#page-24-1) apply.

⁹It is formally defined in subsection [3.1.](#page-10-3)

6 The Issue of Potential Exclusion

In sections [3](#page-10-0) to [5,](#page-19-0) we set conditions ensuring that both types of CPs were always active in the market. In particular, we assumed that the ads revenues were high and equal for both types of CPS. In this Section, we derive some conditions under which some CPs could be forced to exit the market, both in the case of laissez-faire or in the event of a regulation.

6.1 Exclusion with Laissez-Faire

First of all, it should be noted that the question of exclusion only arises if prices are uniform. Indeed, with tailored prices, it is never in the ISP's interest to exclude a CP, since it can always capture the rent generated by the latter. It will simply want to induce the CP to make an effort, as explained in subsection [4.2.](#page-15-1) Therefore, we focus on the case with uniform prices and assume now that the ads revenues $b(\theta) = b$ for types θ and $b(\theta) = b$ for types θ could differ. The question is then whether the ISP decides to set a price compatible with all business models or, on the contrary, to set a price that excludes the less-profitable content.

This potential exclusion would have some impact on the network congestion, and, therefore, on the equilibrium network size. When the less profitable CPs are the capacity-economical CPs $(b < b)$, their exclusion has a limited impact on the load unless they constitute the majority of CPs. Instead, when the less profitable CPs are the capacity-intensive CPs, $(b > b)$, exclusion is more likely to have a positive impact on network congestion and, therefore, induce a fall in the ISP investment level. The following proposition discusses these case.

Proposition 6. When CPs differ in their business models with $b \neq b$,

- *1. to have* $K^u \leq K^n$, *it requires the exclusion of one type of CP.*
- 2. there exist weights $\hat{\mu}$ and μ^* such that for $\hat{\mu} < \mu \leq \mu^*$, ISP will charge a uniform price $p^* = \max{\{\overline{b}, \underline{b}\}}$, and this leads to $K^u \leq K^n$.

The first point states that the reduction of the network size can only be achieved by excluding some CPs. Indeed, the ISP has no means of inducing CPs to exert a congestion-reducing effort. As a consequence, the only way to reduce congestion is to exclude some types of CPs. The second point states some conditions under which the ISP may be willing to exclude some CPs and that this exclusion is good for the environment. The first condition is satisfied when the share of excluded CPs is not too small. For the second, the total capacity needed to serve the remaining CP should not be too large, which means that the share of excluded CPs should be high enough. These two conditions are quite demanding and more likely to be satisfied when the less profitable CPs (those excluded) are also the capacity-intensive CPs. Finally, even if the environmental impact of exclusion is positive, the direct effect on the consumer's gross surplus is likely to be negative. Indeed, consumers will be deprived of one type of content. They will consume more of the remaining content but, with their taste for variety, they are likely to be worse off compared to the net neutrality case.

6.2 Exclusion with Regulation

In Section [5,](#page-19-0) we assumed that all CPs were profitable when they exerted congestionreducing efforts. We now consider alternative cases in which ad revenues are lower than before, and thus may prevent some CPs from exerting effort in a profitable way.

Indeed, if $\psi \overline{\theta} + \big(\psi - \overline{\psi}\big)\hat{e} > b$, capacity-intensive CPs cannot exert a profitable effort with the highest regulated price. Then, if the regulator sets the $p = \psi$, only type- θ CPs will exert congestion-reducing effort, while the other CPs still operate on the market but do not make any effort. In this case, the ISP profit writes as before but the constraint is changed to:

$$
k(E(\theta) - \mu\hat{e}) \le K \tag{6}
$$

The equilibrium consumption and investment level are now given by

$$
u'\left(\underline{k}^r\right) = \left(\mathbb{E}\left(\theta\right) - \mu\hat{e}\right)c \text{ and } \underline{K}^r = \underline{k}^r \left(\mathbb{E}\left(\theta\right) - \mu\hat{e}\right)
$$

As before, when *u* is concave enough, it is direct to show that $\underline{k}^r > k^n$; $\underline{K}^r < K^n$ and $\underline{\Pi} > \Pi^n$.

Moreover, the congestion constraints [\(5\)](#page-22-2) and [\(6\)](#page-27-0) are identical for $\mu = \frac{1}{2}$ $\frac{1}{2}$, i.e., when the share of both types of CPs are the same. So if $\mu \geq \frac{1}{2}$ $\frac{1}{2}$, the total congestion is lower with $p = \psi$ than with $p = \overline{\psi}$ and conversely. This means that setting a high regulated price is still optimal when most of the CPs are θ -type. As a result, we can state the following.

Corollary 6. If $\psi \overline{\theta} + (\psi - \overline{\psi})\hat{e} > b \ge \psi \overline{\theta}$, and there are mostly capacity-economical CPs, *i.e.*, $\mu \geq \frac{1}{2}$ $\frac{1}{2}$, a consumerist-environmentalist regulator will choose $p^r=\underline{\psi}.$ Otherwise, the *regulator will choose* $p^r = \overline{\psi}$ *.*

In this setting, the regulator implements its price regulation in order to obtain the most effective *congestion-based e*ff*ect*. When there are mostly type-θ CPs, it is too harmful for society not to incentivize them. Indeed, in this case, the impact of the regulation on the ISP constraint would be too limited, stalling the congestion-based effect. As a result, the regulator opts for $p^r = \psi$.

With the above price, the $\overline{\theta}$ -types were not excluded from the market. Suppose now that the ad revenues are even lower, and more precisely that $\psi \overline{\theta} > b \ge \max{\psi \theta}$. Then, when $p = \psi$, the type- $\overline{\theta}$ CPs cannot make any positive profit in this market, regardless of their choice of effort. If those CPs are excluded from the market, the ISP problem changes to

$$
\max_{k,K} \Pi = \mu u(k) - cK \text{ s.t. } k\mu\left(\underline{\theta} - \hat{e}\right) \le K
$$

Then, the equilibrium consumption and investment levels are now given by

$$
u'\left(\underline{k}^r\right) = \left(\underline{\theta} - \hat{e}\right)c \text{ and } \underline{K}^r = \underline{k}^r \mu\left(\underline{\theta} - \hat{e}\right)
$$
 (7)

And, for a sufficiently concave *u*, we still obtain $\underline{k}^r > \overline{k}^r > k^n$ and $\underline{K}^r < \overline{K}^r < K^n$.

The optimal choice of regulatory price by a C-E regulator is now based on the welfare function $\hat{W}_e(p) = \mu(p)u(k) - (\delta + c)K$, with $\mu(\psi) = \mu$ and $\mu(\overline{\psi}) = 1$. Then, there exists a threshold μ^r such that $\hat{W}_e\big(\psi\big)\geq W_e\big(\overline{\psi}\big)$ if $\mu\geq\mu^r.$

Corollary 7. If $\psi \overline{\theta} > b \ge \max{\psi \theta}$, and there are mainly type- θ CPs ($\mu \ge \mu^r$), a C-E *regulator will choose* $p^r = \psi$ *, and conversely.*

This result is reminiscent of Corollary [6.](#page-28-0) Indeed, the consumerist-environmentalist regulator will choose a price that allows the most common CPs to exert some

congestion-reducing effort. This shows that even a low regulated price can be welfare optimal, as long as it induces most of the CPs to exert some effort.

Finally, let us consider the lowest admissible values for the ad revenues, i.e., the case in which max $_{\theta}$ { ψ (θ) θ } > $b \ge \min_{\theta}$ { ψ (θ) θ }. Now, the exclusion of CPs can occur for the two possible regulated price levels. Indeed, suppose first that $\psi \theta > \psi \theta$. Then, setting a regulated price $p^r = \psi$ will exclude all CPs so it is not feasible and the only possible policy is to set $p^r = \overline{\psi}$. Suppose instead that $\overline{\psi}\theta > \psi \underline{\theta}$. Then, for both prices $p^r = \psi$ or $p^r = \overline{\psi}$, the θ -type CPs will be excluded. However, the regulator wants to induce the remaining CPs to exert some effort so it will choose $p^r = \psi$ and obtains $\hat{W}_e(\psi) = \mu u\left(\underline{k}^r\right) - (\delta + c)\,\underline{K}^r$ where the consumption and investment levels are defined in [\(7\)](#page-28-1).

7 Conclusions

For years, the debate about the regulation of the Internet has been limited to actors in the industry, mostly Internet service providers and content providers. Recent awareness of the industry's environmental footprint has made this question much more global. It has also changed the perspective one should have on the issue at stake; whereas the original question was, "how to provide the best incentives to increase the size of the network", it has become, "how to provide the right incentives to limit the environmental impact of the sector". In this respect, we have shown that the current situation is far from optimal. The lack of both regulation and prices between CPs and ISPs leads those players, as well as consumers, to overlook the negative externalities their activity generates on the environment. The freedom that was gained thanks to the implementation of net neutrality has created a moral hazard issue whose consequences can be measured by a sharp rise in CO2 emissions linked to the Internet.

We presented some possible remedies to this issue that all point to the need for more incentives for the players that can reduce the need for a larger network. In particular, we showed what a regulator could do by setting prices to induce CPs to account for their impact on the environment. This solution relies on the idea that consumers could benefit from the same quality of service at a lower cost for the industry. Introducing some prices on the supply side would allow more coordination and lead to a lower negative environmental impact of the Internet.

Another possibility that we do not explore in this article would be to influence the demand side. For example, allowing ISPs to propose some plans with limited load would be useful to smooth total consumption and limit the need for network expansion. According to [Liu et al.](#page-32-2) [\(2019\)](#page-32-2), there are "big points" of sustainable ICT consumption (from applications to videostreaming) and options for educating consumers that must be explored further in order to strengthen the users' capacities with regard to a sustainable digital consumption. For instance, [Obringer et al.](#page-32-0) [\(2021\)](#page-32-0) shows that if some digital sobriety habits could be adopted by users or proposed by the telecom operators, some substantial environmental gains could be achieved. As an example, lowering the video quality of streaming services for 70 million of subscribers would lead to a monthly reduction approximately equivalent of 6% of the total monthly coal consumption in the US. It is difficult to know whether it would be more efficient to play on the supply side or on the demand side. In either case, a move from the current situation of net neutrality is needed to seriously tackle the environmental impact of the digital and telecom industries. This should not be interpreted by policymakers as a plea for less regulation but as a recommendation to incorporate environmental concerns in the evaluation of regulatory practices.

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8 Appendix

A useful Lemma

In the different proofs above, we will need to invoke the following useful intermediate result.

Lemma A. *The solution in K of the following equation where* x *,* y *,* $a > 0$ *:*

$$
u'\left(\frac{K}{ax}\right) = cx - y
$$

is an implicit function is $\mathcal{K}\left(x,y\right)$ such that

- 1. $\mathcal{K}'_x(x,y) > 0$ for all a, *y* if *u* is concave enough such that $\rho(\mathcal{K}/ax) > \frac{cx}{cx-y} \ge 1$ where $\rho(k) = -\frac{u''(k)k}{u'(k)}$ $\frac{u^{(k)}(k)}{u^{(k)}} > 0.$
- 2. $\mathcal{K}'_y(x, y) > 0$ for all a, x

Proof of Lemma [A.](#page-33-0) First, differentiating with respect to $x \ge 0$ leads to

$$
u''\left(\frac{\mathcal{K}}{ax}\right)\left(\frac{1}{a}\frac{\mathcal{K}'_x x - \mathcal{K}}{x^2}\right) = c > 0
$$

so

$$
\mathcal{K}'_x = a \frac{cx}{u''\left(\mathcal{K}/x\right)} + \frac{\mathcal{K}}{x}
$$

which can be rearranged as:

$$
\mathcal{K}'_x = a \frac{cx}{u''(\mathcal{K}/ax)} + \frac{\mathcal{K}}{x}
$$

\n
$$
= a \frac{u'(\mathcal{K}/ax) + y}{u''(\mathcal{K}/ax)} + a \frac{\mathcal{K}}{ax}
$$

\n
$$
= \frac{\mathcal{K}}{x} \left[1 + \frac{u'(\mathcal{K}/ax) + y}{\frac{\mathcal{K}}{ax}u''(\mathcal{K}/ax)} \right]
$$

\n
$$
= a \mathcal{K} \left(1 - \frac{1}{\rho(\mathcal{K}/ax)} \frac{u'(\mathcal{K}/ax) + y}{u'(\mathcal{K}/ax)} \right)
$$

where ρ is an "Arrow–Pratt of relative risk aversion"-like measure or a relative curvature index also known as the elasticity of slope of the utility; here, there is no uncertainty but heterogeneity, so ρ is a heterogeneity elasticity for consumers

$$
\rho(k) = -\frac{u''(k)k}{u'(k)} > 0
$$

So, we have

$$
\rho\left(\mathcal{K}/ax\right) > \frac{u'\left(\mathcal{K}/ax\right) + y}{u'\left(\mathcal{K}/ax\right)} \ge 1 \Rightarrow \mathcal{K}'_x > 0 \text{ for all } y \ge 0
$$

This means that *u* (*k*) is sufficiently concave. If ρ (*K*/*ax*) < 1, then *K*_{*x*} < 0, i.e., *u* (*k*) is not too concave

Second, differentiating with respect to $y \geq 0$ leads to

$$
\mathcal{K}'_y u''\left(\frac{\mathcal{K}}{ax}\right) = -ax < 0
$$

so $\mathcal{K}'_y > 0$ for all $x \ge 0$. Last, differentiating with respect to $1 \ge a \ge 0$ leads to

$$
\mathcal{K}'_a=\frac{\mathcal{K}}{a}>0
$$

Proof of Proposition [1](#page-10-1)

In order to lighten the notations, we denote any variable $x(\underline{\theta}) = \underline{x}$ and $x(\overline{\theta}) = \overline{x}$. Let the Lagrangian $L = W + \lambda \left(K - \mu\left(\underline{\theta} - \underline{e}\right)q - (1 - \mu)\left(\overline{\theta} - \overline{e}\right)\overline{q}\right)$ where

$$
W = \mu \left[u \left(\underline{q} \right) + \left(\underline{b} - \underline{\psi} \underline{e} \right) \underline{q} \right] + (1 - \mu) \left[u \left(\overline{q} \right) + \left(\overline{b} - \overline{\psi} \overline{e} \right) \overline{q} \right] - \delta K - C(K)
$$

Khun-Tucker conditions yield

$$
u'(\underline{q}) + b - \underline{\psi} \underline{e} = \lambda (\underline{\theta} - \underline{e})
$$

$$
u'(\overline{q}) + b - \overline{\psi} \overline{e} = \lambda (\overline{\theta} - \overline{e})
$$

$$
\lambda > \underline{\psi} > \overline{\psi} \implies \underline{e} = \overline{e} = \hat{e}
$$

$$
\underline{\psi} > \lambda > \overline{\psi} \implies \underline{e} = 0 \text{ and } \overline{e} = \hat{e}
$$

$$
\underline{\psi} > \overline{\psi} > \lambda \implies \underline{e} = \overline{e} = 0
$$

$$
\lambda = \delta + c > 0
$$

As $\lambda > 0$ then $K = \mu \left(\frac{\theta}{2} - \frac{e}{2} \right) q + (1 - \mu) \left(\overline{\theta} - \overline{e} \right) \overline{q}.$

• If $\underline{\psi} > \overline{\psi} > \delta + c \Rightarrow \underline{e} = \overline{e} = 0$, and:

$$
u' \left(\frac{q}{q}\right) = (\delta + c) \underbrace{\theta - b}_{\text{u'}} u' \left(\overline{q}\right) = (\delta + c) \overline{\theta} - \overline{b} \underline{q}_0 > \overline{q}_0
$$

then $K_0^* = \mu \underline{\theta} \underline{q}_0 + (1 - \mu) \overline{\theta} \overline{q}_0$.

• If $\underline{\psi} > \delta + c > \overline{\psi} \Rightarrow \underline{e} = 0$ and $\overline{e} = \hat{e}$, and:

$$
u' \left(\frac{q}{q_0}\right) = (\delta + c) \underline{\theta} - \underline{b}
$$

$$
u' \left(\overline{q}_1\right) = (\delta + c) \overline{\theta} + \left(\overline{\psi} - (\delta + c)\right) \hat{e} - \overline{b}
$$

$$
\overline{q}_1 > \overline{q}_0
$$

then $K_{01}^* = \mu \underline{\theta} \underline{q}_0 + (1 - \mu) \left(\overline{\theta} - \hat{e} \right) \overline{q}_1$.

• If $\delta + c > \underline{\psi} > \overline{\psi} \Rightarrow \underline{e} = \overline{e} = \hat{e}$, and:

$$
u'\left(\underline{q}_{1}\right) = (\delta + c)\underline{\theta} + \left(\underline{\psi} - (\delta + c)\right)\hat{e} - \underline{b}
$$

$$
u'\left(\overline{q}_{1}\right) = (\delta + c)\overline{\theta} + \left(\overline{\psi} - (\delta + c)\right)\hat{e} - \overline{b}
$$

$$
u'_{1} = \mu\left(\underline{\theta} - \hat{e}\right)\underline{q}_{1} + (1 - \mu)\left(\overline{\theta} - \hat{e}\right)\overline{q}_{1}.
$$

Proof of Proposition [2](#page-13-1)

then *K* ∗

The ISP problem writes as

$$
\max_{(k,K)} \Pi = u(k) - C(K) \text{ s.t. } k \mathbb{E}(\theta) \le K
$$

then as $\frac{\partial \Pi}{\partial k} = u'(k) > 0$ and $\frac{\partial \Pi}{\partial k} = -c < 0$, the constraint is binding and

 $q^{n}(\theta) = k^{n} : u'(k^{n}) = c \mathbb{E}(\theta)$ and $K^{n} = k^{n} \mathbb{E}(\theta)$

Proof of Proposition [3](#page-14-0)

With $b(\overline{\theta}) = b(\underline{\theta}) = b$, the CP profit is $\pi(\theta) = (b - p)k \ge 0$. So, the ISP problem is then

$$
\max_{(p,k,K)} u(k) + pk - C(K) \text{ s.t. } k \mathbb{E}(\theta) \le K \text{ and } b \ge p
$$

then

$$
p^{u} = b
$$

\n
$$
q^{u}(\theta) = k^{u} = \frac{K^{u}}{\mathbb{E}(\theta)}
$$

\n
$$
K^{u} : u'(\frac{K^{u}}{\mathbb{E}(\theta)}) = c\mathbb{E}(\theta) - b < c\mathbb{E}(\theta) = u'(\frac{K^{n}}{\mathbb{E}(\theta)})
$$

so by concavity of *u*, this leads to $\frac{K^u}{\mathbb{E}(\theta)} > \frac{K^n}{\mathbb{E}(\theta)}$ $\frac{\kappa^{\alpha}}{\mathbb{E}(\theta)}$, and we always have:

 $k^n < k^u$ and $K^n < K^u$

Proof of Lemma [1](#page-15-2)

For tariff $t(\theta)$, $p(\theta)$, the ISP profit writes as

$$
\Pi = u(k) + \mu \left(t(\underline{\theta}) + p(\underline{\theta})(\underline{\theta} - e(\underline{\theta})) \right) + (1 - \mu) \left(t(\overline{\theta}) + p(\overline{\theta})(\overline{\theta} - e(\overline{\theta})) \right) k - cK
$$

under the congestion constraint. In this case, the ISP will capture the whole CP surplus. Therefore, for any *e*, we have $t = b - p(\theta - e) - \psi e$. Moreover, *e* depends on *p*. The ISP must choose *k*,*K* and the price *p* that pins down the effort chosen by the CP. The ISP profit now writes as

$$
\Pi = u(k) + \mu \left(b - \underline{\psi}e(\underline{\theta}) \right) k + (1 - \mu) \left(b - \overline{\psi}e(\overline{\theta}) \right) k - cK
$$

subject to $\mu\left(\frac{\theta}{\theta} - e(\frac{\theta}{\theta})\right)k + (1 - \mu)\left(\overline{\theta} - e(\overline{\theta})\right)k \leq K$.

Optimizing wrt to $p(\underline{\theta})$ leads to $\frac{de}{dp}(-\psi + \lambda)\mu k$. It is direct to see, optimizing with respect to *K*, that the multiplier associated with the congestion constraint is $\lambda = c$. Therefore, the sign of the derivative will depend on $c - \psi$ (and similarly $c - \overline{\psi}$ for the $\overline{\theta}$ -CPs). Since $\frac{de}{dp} \ge 0$, the result follows directly.

Proof of Proposition [4](#page-15-0) and Corollary [2](#page-17-0)

If $b > \theta \psi(\theta)$, for all θ , then the ISP profit writes as $\Pi = u(k) + \mathbb{E}(b - \theta \psi(\theta))k - cK$, so the problem is

$$
\max_{k,K} \Pi \text{ s.t. } k(\mathbb{E}(\theta) - \hat{e}) \leq K
$$

and equilibrium is given by

$$
u'\left(k^{t}\right) = c\left(\mathbb{E}\left(\theta\right) - \hat{e}\right) - \mathbb{E}\left(b - \theta\psi(\theta)\right)
$$

$$
K^{t} = \left(\mathbb{E}\left(\theta\right) - \hat{e}\right)k^{t}
$$

If *u* is concave enough by invoking Lemma [A,](#page-33-0) we have

$$
\mathcal{K}(\mathbb{E}(\theta) - \hat{e}, \mathbb{E}(b - \theta \psi(\theta))) = K^{t} \text{ and } K^{n} = \mathcal{K}(\mathbb{E}(\theta), 0)
$$

Then

$$
K^{n}=\mathcal{K}\left(\mathbb{E}\left(\theta\right),0\right)>\mathcal{K}(\mathbb{E}\left(\theta\right)-\hat{e},0)
$$

and since $\mathbb{E}\left(b - \theta \psi(\theta)\right) > 0$ as $\mathcal{K}'_y(x, y) > 0$:

$$
\mathcal{K}\left(\mathbb{E}\left(\theta\right)-\hat{e},0\right)<\mathcal{K}\left(\mathbb{E}\left(\theta\right)-\hat{e},\mathbb{E}\left(b-\theta\psi(\theta)\right)\right)=K^{t}
$$

Hence, as there exists a level \overline{B} of $\mathbb{E}\left(b-\theta\psi(\theta)\right)$ such that $K^t\leq K^n$ when $\mathbb{E}\left(b-\theta\psi(\theta)\right)\leq K^n$ *B*. More precisely,

$$
\overline{B} = c \left(\mathbb{E} \left(\theta \right) - \hat{e} \right) - u' \left(\frac{k^n \mathbb{E} \left(\theta \right)}{\mathbb{E} \left(\theta \right) - \hat{e}} \right).
$$

This proves Proposition [4.](#page-15-0)

To prove Corollary [2,](#page-17-0) we see that, for any $e \leq \hat{e}$

$$
\frac{\partial K^t}{\partial e} = \frac{\partial \mathcal{K}(x,y)}{\partial e} = -\mathcal{K}'_x(x,y)
$$

where $x = \mathbb{E}(\theta) - e$ and $y = \mathbb{E}(b - \theta \psi(\theta))$. From Lemma [A,](#page-33-0) we have directly ∂K(*x*,*y*) $\frac{\partial (x,y)}{\partial e}$ < 0.

Proof of Corollary [1](#page-16-0)

Indeed, if for *only one* given CP θ , $b - \psi(\theta) \theta > 0$, and

$$
k^{\tau} : u' (k^d) = c \mathbb{E}(\theta) - cm(\theta) \hat{e} - (b - \theta \psi(\theta)) < cm(\theta) \hat{e}
$$

$$
K^{\tau} = (\mathbb{E}(\theta) - m(\theta) \hat{e}) k^{\tau}
$$

Invoking Lemma [A,](#page-33-0) we have $K^{\tau} = \mathcal{K}(\mathbb{E}(\theta) - m(\theta)\hat{e}, b - \theta\psi(\theta)) > \mathcal{K}(\mathbb{E}(\theta) - m(\theta)\hat{e}, 0)$ but $\mathcal{K}(\mathbb{E}(\theta) - m(\theta)\hat{\ell},0) < \mathcal{K}(\mathbb{E}(\theta),0) = K^n$. Hence, it exists a level <u>B</u> such that $K^{\tau} = (\mathbb{E}(\theta) - m(\theta)\hat{e})k^{\tau} \leq K^{n}$ when $b - \theta\psi(\theta) \leq \underline{B}$. Then

$$
\underline{B} = (\mathbb{E}(\theta) - \hat{e})c - u' \left(k^n \frac{\mathbb{E}(\theta)}{\mathbb{E}(\theta) - m(\theta)\hat{e}} \right).
$$

Proof of Proposition [5](#page-22-0) and Corollaries

For Proposition [5](#page-22-0). (a) If $p = \psi > \overline{\psi}$ then

$$
u'\left(\underline{k}^r\right) = c\left(\mathbb{E}\left(\theta\right) - \hat{e}\right) \quad \text{and} \quad \underline{K}^r = \underline{k}^r\left(\mathbb{E}\left(\theta\right) - \hat{e}\right) \tag{8}
$$

so as c ($\mathbb{E}(\theta) - \hat{e}$) < $c \mathbb{E}(\theta)$ by concavity of $\underline{k}^r > k^n$. **(b)** If $\psi > p = \overline{\psi}$ then

$$
u'\left(\overline{k}'\right) = \left(\mathbb{E}\left(\theta\right) - \left(1 - \mu\right)\hat{e}\right)c \text{ and } \overline{K}' = \overline{k}'\left(\mathbb{E}\left(\theta\right) - \left(1 - \mu\right)\hat{e}\right)
$$

Straightforwardly: $k^n < \overline{k}^r$.

(c) Invoking Lemma [A,](#page-33-0) when *u* is concave enough, we have

$$
\underline{K}^r = \mathcal{K}(\mathbb{E}(\theta) - \hat{e}, 0) < \overline{K}^r = \mathcal{K}(\mathbb{E}(\theta) - (1 - \mu)\hat{e}, 0) < K^n = \mathcal{K}(\mathbb{E}(\theta), 0)
$$

Moreover, $\underline{k}^r > \overline{k}^r$ as

$$
u'\left(\underline{k}^r\right) = c\left(\mathbb{E}\left(\theta\right) - \hat{e}\right) < \left(\mathbb{E}\left(\theta\right) - \left(1 - \mu\right)\hat{e}\right)c = u'\left(\overline{k}^r\right)
$$

Consequently,

$$
\underline{\Pi} = u\left(\underline{k}^r\right) - c\underline{K}^r > \overline{\Pi} = u\left(\overline{k}^r\right) - c\underline{K}^r > \Pi^n = u\left(k^n\right) - cK^n
$$

For the Corollary [3.](#page-23-0) From [\(3\)](#page-11-0), we define $W_e(p) = u(k) - (\delta + c)K$ and using results in Proposition [5,](#page-22-0) we get:

$$
W_e(\underline{\psi}) = u(\underline{k}') - (\delta + c) \underline{K}' > u(\overline{k}') - (\delta + c) \underline{K}' > u(\overline{k}') - (\delta + c) \overline{K}' = W_e(\overline{\psi})
$$

so $p^r = \psi$ is preferred by the regulator.

For the Corollary [6.](#page-28-0) As when $\mu \geq \frac{1}{2}$ we have $\underline{k}^r \geq \overline{k}^r$ and $\underline{K}^r \leq \overline{K}^r$

$$
W_e(\underline{\psi}) = u(\underline{k}') - (\delta + c) \underline{K}' \ge u(\overline{k}') - (\delta + c) \overline{K}' = W_e(\overline{\psi})
$$

so $p^r = \psi$ is preferred by the regulator. Conversely, when $\mu < \frac{1}{2}$. **For the Corollary** [7.](#page-28-2) As now invoking Lemma [A,](#page-33-0) $\underline{K}^r = \mathcal{K}(\mu \underline{\theta} - \mu \hat{e}, 0) < \overline{K}^r =$ $\mathcal{K}(\mathbb{E}(\theta) - (1 - \mu)\hat{e}, 0)$ as

$$
\mu \underline{\theta} - \mu \hat{e} < \mu \underline{\theta} \leq \mathbb{E}(\theta) - (1 - \mu) = \mu \underline{\theta} + (1 - \mu) \left(\overline{\theta} - \hat{e} \right)
$$

then

$$
\hat{W}_e(\underline{\psi}) = \mu u(\underline{k}^r) - (\delta + c) \underline{K}^r \ge u(\overline{k}^r) - (\delta + c) \overline{K}^r = W_e(\overline{\psi})
$$

if

$$
\mu \ge \mu^r = \frac{u\left(\overline{k}^r\right) - (\delta + c)\left(\overline{K}^r - \underline{K}^r\right)}{u\left(\underline{k}^r\right)} < 1 - (\delta + c)\frac{\overline{K}^r - \underline{K}^r}{u\left(\underline{k}^r\right)}
$$

Proof of Corollary [4](#page-24-0)

First note that the gross consumers' surplus derived from consumption is $u(k)$ a montonone increasing function of *k*. Then ranking any values of *k* provide a ranking for *u*. From our results in the text we have

$$
k^n : u'(k^n) = \mathbb{E}(\theta) c
$$

\n
$$
k^u : u'(k^u) = \mathbb{E}(\theta) c - b
$$

\n
$$
k^t : u'(k^t) = (\mathbb{E}(\theta) - \hat{e})c - \mathbb{E}(b - \theta \psi(\theta))
$$

\n
$$
\underline{k}^r : u'(\underline{k}^r) = (\mathbb{E}(\theta) - \hat{e})c
$$

Then by concavity of *u*, one can assess

$$
\mathbb{E}(\theta)c > (\mathbb{E}(\theta) - \hat{e})c > (\mathbb{E}(\theta) - \hat{e})c - \mathbb{E}(b - \theta\psi(\theta)) \Rightarrow k^n < \underline{k}^r < k^t
$$

This allows us to provide the following ranking

$$
k^{n} < \underline{k}^{r} < k^{t} \le k^{u} : \text{if } \mathbb{E}(\theta \psi(\theta)) \ge \hat{e}c
$$
\n
$$
k^{n} < \underline{k}^{r} \le k^{u} < k^{t} : \text{if } b \ge \hat{e}c > \mathbb{E}(\theta \psi(\theta))
$$
\n
$$
k^{n} < k^{u} < \underline{k}^{r} < k^{t} : \text{if } \hat{e}c > b
$$

Second, due to the ISP pricing we considered, note that the net consumers' surplus is $u(k) - T = 0$ for all *k* while the net indirect consumers' utility is $U = -\delta K$, the environmental damage. Now based on the comparison of consumption levels above, we have:

$$
K^{t} = (\mathbb{E}(\theta) - \hat{e}) k^{t} > \underline{K}^{r} = (\mathbb{E}(\theta) - \hat{e}) \underline{k}^{r}
$$

Proof of Corollary [5](#page-24-1)

From [\(3\)](#page-11-0) one can define consumerist-environmentalist welfare levels for respectively the Net Neutrality, Uniform Prices, Tailored Prices and Regulated Prices outcomes.

$$
W_e^n = u(k^n) - (\delta + c) K^n = u(k^n) - (\delta + c) \mathbb{E}(\theta) k^n
$$

\n
$$
W_e^u = u(k^u) - (\delta + c) K^u = u(k^u) - (\delta + c) \mathbb{E}(\theta) k^u
$$

\n
$$
W_e^t = u(k^t) - (\delta + c) K^t = u(k^t) - (\delta + c) (\mathbb{E}(\theta) - \hat{e}) k^t
$$

\n
$$
W_e^r = u(\underline{k}^r) - (\delta + c) K^r = u(\underline{k}^r) - (\delta + c) (\mathbb{E}(\theta) - \hat{e}) \underline{k}^r
$$

To compare them, let us define two functions

- 1. $\Omega(k) = u(k) (\delta + c) \mathbb{E}(\theta) k$ achieves a maximum in $k^{\omega} : u'(k^{\omega}) = (\delta + c) \mathbb{E}(\theta)$ with $k^{\omega} < k^{n} < k^{\mu}$, so directly we have $W_{e}^{n} = \Omega(k^{n}) > \Omega(k^{\mu}) = W_{e}^{u}$
- 2. $\hat{\Omega}(k) = u(k) (\delta + c)(\mathbb{E}(\theta) \hat{e})k$ achieves a maximum in $\hat{k}^{\omega}: u'(k) = (\delta + c)(\mathbb{E}(\theta) \hat{e})$ with $k^{\omega} < \hat{k}^{\omega} < k^t < k^t$ and directly we have $W^r_e = \hat{\Omega}\left(\underline{k}^r\right) > \hat{\Omega}\left(k^t\right) = W^t_e.$

So With regards to the C-E welfare criterion:

- Net Neutrality is preferred to laissez-faire with uniform prices
- Price regulation is preferred to laissez-faire with tailored prices

First if *u* is concave enough, we know from our propositions [2](#page-13-1) to [5](#page-22-0) that one can have one of the following rankings for the network capacity, where K' is the lowest:

1. $K^u > K^n \geq K^t > K^r$ and from the elements of proof of the Corollary [4](#page-24-0) we have straightforwardly the following total ordering:

$$
W_e^r > W_e^t = u(k^t) - (\delta + c) K^t > W_e^n = u(k^n) - (\delta + c) K^n > W_e^n
$$

Price regulation is the most preferred regime.

2. One can also have $K^u > K^t > K^n > K^r$ which leads to the following partially ambiguous ranking

 $W_e^r > W_e^n > W_e^u$ but $W_e^n \leq W_e^t$

However, price regulation is still the most preferred regime.

Second, if *u* is not concave enough then we know from our propositions [2](#page-13-1) to [5](#page-22-0) that, depending on the weakness of the curvature of the utility function, one can have one of the following rankings for the network capacity, where K^n is the lowest.

1. $K^t > K^n > K^n$ which leads to a total ordering:

$$
\text{if } b \geq \hat{e}c: W_e^n > W_e^u > W_e^r > W_e^t
$$

So with regards to the EC criterion, when $b \geq \hat{c}c$, Net Neutrality is now the most preferred regime. However if $\hat{e}c > b$ then we have $W_e^u \leq W_e^r$ but the previous result still holds.

2. $K^t > K^u > K^r > K^n$. Then if $\hat{e}c > b$, it leads to the following partially ambiguous ranking

$$
W_e^n > W_e^u
$$
 and $W_e^r > W_e^t$ but $W_e^u \leq W_e^r$

However the previous result still holds. If $b \geq \hat{c}c$, we have the following global ordering: $W_e^n > W_e^r \geq W_e^u > W_e^t$, such that the previous result still holds.

Proof of Proposition [6](#page-26-1)

When $\bar{b} > \underline{b}$. So, if $p^* = \underline{b}$ no exclusion occurs and the ISP problem is then

$$
\max_{(k,K)} u(k) + \underline{b}k - C(K) \quad \text{s.t. } k \mathbb{E}(\theta) \le K
$$

then

$$
q^{u}(\theta) = \underline{k}^{u} = \frac{\underline{K}^{u}}{\mathbb{E}(\theta)}
$$

$$
\underline{K}^{u} : u'\left(\frac{\underline{K}^{u}}{\mathbb{E}(\theta)}\right) = c\mathbb{E}(\theta) - \underline{b}
$$

If so, if $p^* = \overline{b}$, CP <u> θ </u> is excluded and then

$$
\max_{(k,K)} (1 - \mu) \left[u(k) + \overline{b}k \right] - C(K) \text{ s.t. } k(1 - \mu) \overline{\theta} \le K
$$

then

$$
\underline{q}^{u} = 0 \text{ and } \overline{q}^{u} = \overline{k}^{u} = \frac{\overline{K}^{u}}{(1 - \mu)\overline{\theta}}
$$

$$
\overline{K}^{u} : u'\left(\frac{K^{u}}{(1 - \mu)\overline{\theta}}\right) = c\overline{\theta} - \overline{b}
$$

and we see that we always have:

$$
u' \left(\frac{K^n}{\mathbb{E}(\theta)} \right) = c \mathbb{E}(\theta) > u' \left(\frac{\underline{K}^u}{\mathbb{E}(\theta)} \right) = c \mathbb{E}(\theta) - \underline{b}
$$

$$
\Rightarrow k^n < \underline{k}^u \text{ and } K^n < \underline{K}^u
$$

Therefore, it is not possible to reduce capacity without exclusion, that is, $p^* = \overline{b} =$ $max{\{\overline{b}, \underline{b}\}}$ is needed, which proves Part 1 of the Proposition when $\overline{b} > \underline{b}$.

To prove Part 2 of the Proposition (when $\bar{b} > b$), we have to determine if it is optimal for the ISP to adopt $p^* = \overline{b}$. Optimal levels of profit are such that

$$
\Delta\Pi(\mu) = \overline{\Pi}^{u}(\mu) - \underline{\Pi}^{u}(\mu)
$$

\n
$$
\Rightarrow \Delta\Pi(\mu) = (1 - \mu) \left[u(\overline{k}^{u}) - (c\overline{\theta} - \overline{b}) \overline{k}^{u} \right] - \left[u(k^{u}) - (cE(\theta) - \underline{b})k^{u} \right] \ge 0
$$

\n
$$
\Rightarrow \Delta\Pi(\mu) = (1 - \mu) V(\overline{k}^{u}) - V(\underline{k}^{u}) \ge 0
$$

where $V(k) = u(k) - u'(k)k$ is an increasing function of *k* as $V'(k) = -u''(k)k < 0$ by concavity of *u*. So, if $\underline{k}^u \ge \overline{k}^u$ i.e., if $0 < \overline{b} - \underline{b} \le c(\overline{\theta} - \mathbb{E}(\theta)) \Delta \Pi(\mu) \le 0$, and the ISP cannot choose $p^* = \overline{b}$. Hence, if $\overline{b} - \underline{b} > c \left(\overline{\theta} - \mathbb{E} \left(\theta \right) \right) \Leftrightarrow \mu < \underline{\mu} = \frac{\overline{b} - \underline{b}}{c \Delta(\theta)}$ we have

$$
\overline{k}^{u} = \frac{\overline{K}^{u}}{(1-\mu)\overline{\theta}} > \frac{\underline{K}^{u}}{\mathbb{E}(\theta)} = \underline{k}^{u} \Rightarrow \overline{K}^{u} > \frac{(1-\mu)\overline{\theta}}{\mathbb{E}(\theta)} \underline{K}^{u}
$$

So, when $\overline{k}^u > \underline{k}^u$, we have:

$$
\Delta\Pi' \left(\mu \right) = -V \left(\overline{k}^u \right) + c \left(\overline{\theta} - \underline{\theta} \right) \underline{k}^u \le - \left[u \left(\overline{k}^u \right) - \left(c \underline{\theta} - \overline{b} \right) \overline{k}^u \right] < 0
$$

so ΔΠ(μ) ≤ 0 for all μ and ΔΠ(0) = $V(\bar{k}^u) - V(\underline{k}^u) > 0$. So, it exists $μ^*$: ΔΠ(μ^{*}) = 0, such that $\mu \le \mu^* < \mu$, ΔΠ(μ) ≥ 0. To have K^u ≤ K^n we need to verify \bar{b} ≥ $c(\overline{\theta} - \mathbb{E}(\theta)) \Leftrightarrow \mu \leq \overline{\mu} = \frac{\overline{b}}{c\Delta\theta}$ $\frac{b}{c\Delta(\theta)}$. Now when $\mu < \mu < \overline{\mu}$:

$$
\overline{K}^{u} \leq K^{n} \Leftrightarrow (1 - \mu) \overline{\theta K}^{u} \leq \mathbb{E}(\theta) K^{n}
$$

$$
K^{n} = \mathbb{E}(\theta) \gamma (\mathcal{C} \mathbb{E}(\theta)) \geq \overline{K}^{u} = (1 - \mu) \overline{\theta} \gamma (\mathcal{C} \overline{\theta} - \overline{b})
$$

where $\gamma = (u')^{-1}$. Let us form $G(\mu) = (\overline{\theta} - \mu \Delta(\theta)) \gamma (\overline{\epsilon} \overline{\theta} - \mu \epsilon \Delta(\theta))$ with

$$
G(0) = \overline{\theta}\gamma\left(c\overline{\theta}\right) < \overline{\theta}\gamma\left(c\overline{\theta} - \overline{b}\right)
$$

$$
G(1) = \underline{\theta}\gamma\left(c\underline{\theta}\right) > 0
$$

Then it exists $\hat{\mu}: G(\hat{\mu})=(1-\hat{\mu})\overline{\theta}\gamma(c\overline{\theta}-\overline{b})$, such that $\overline{K}^u\leq K^n$ iff $\mu\geq\hat{\mu}$. So, whenever $\hat{\mu} < \mu^*$ when $\mu \in [\hat{\mu}, \mu^*]$, we have the result that $\Delta \Pi(\mu) \geq 0$ and $\underline{K}^u \leq K^n$.

When $\underline{b} > \overline{b}$. So, if $p^* = \overline{b}$ no exclusion arises and the ISP problem is then

$$
\max_{(k,K)} u(k) + \overline{b}k - cK \text{ s.t. } k \mathbb{E}(\theta) \le K
$$

then

$$
q^{u}(\theta) = \vec{k}^{u} = \frac{\overline{K}^{u}}{\mathbb{E}(\theta)}
$$

$$
\overline{K}^{u} : u'(\frac{K^{u}}{\mathbb{E}(\theta)}) = c\mathbb{E}(\theta) - \overline{b}
$$

If so, if $p^* = \underline{b}$, CP $\overline{\theta}$ is excluded and then

$$
\max_{(k,K)} \mu \left[u(k) + \underline{b}k \right] - cK \text{ s.t. } k\mu \underline{\theta} \le K
$$

then

$$
\overline{q}^{u} = 0 \text{ and } \underline{q}^{u} = \underline{k}^{u} = \frac{\underline{K}^{u}}{\mu \underline{\theta}}
$$

$$
\underline{K}^{u} : u' \left(\frac{\underline{K}^{u}}{\mu \underline{\theta}}\right) = c\underline{\theta} - \underline{b}
$$

As a result,

$$
\underline{k}^{u} = \frac{\underline{K}^{u}}{\mu \underline{\theta}} > \frac{\overline{K}^{u}}{\mathbb{E}(\theta)} = \overline{k}^{u} \text{ if } \underline{b} > \overline{b} - c \left(\mathbb{E}(\theta) - \underline{\theta} \right)
$$
\n
$$
\Rightarrow \underline{K}^{u} > \frac{\mu \underline{\theta}}{\mathbb{E}(\theta)} \overline{K}^{u}
$$

and we see that we always have:

$$
u' \left(\frac{K^n}{\mathbb{E}(\theta)} \right) = c \mathbb{E}(\theta) > u' \left(\frac{\overline{K}^u}{\mathbb{E}(\theta)} \right) = c \mathbb{E}(\theta) - \overline{b}
$$

$$
\Rightarrow \underline{k}^n < \overline{k}^u \text{ and } \underline{K}^n < \underline{K}^u
$$

Therefore, it is not possible to reduce the capacity without exclusion, that is, $p^* = \underline{b}$ max{*b*, *b*}, which finishes to prove Part 1 of the proposition. To complete the proof of Part 2 of the Proposition (when $\bar{b} < \underline{b}$), we must determine if it is optimal for the ISP to adopt $p^* = b$, when optimal levels of profit are such that

$$
\underline{\Pi}^{u}(\mu) - \overline{\Pi}^{u}(\mu) \iff \mu \left[u \left(\underline{k}^{u} \right) - \left(c \underline{\theta} - \underline{b} \right) \underline{k}^{u} \right] - \left[u \left(\overline{k}^{u} \right) - \left(c \mathbb{E} \left(\theta \right) - \overline{b} \right) \overline{k}^{u} \right] \ge 0
$$

$$
\iff \mu V \left(\underline{k}^{u} \right) - V \left(\overline{k}^{u} \right) \ge 0
$$

So, if $\mu \geq \mu^*$ such that $\mu^* = \frac{V(\underline{k}^u)}{V(\overline{k}^u)}$ $\frac{\partial^2 u}{\partial(\vec{k})^u}$. Moreover, u' $\left(\frac{K^n}{\sqrt{n}} \right)$ $\overline{\mathbb{E}(\theta)}$! $> u' \left(\frac{\underline{K}^u}{\mu \underline{\theta}} \right) = c \underline{\theta} - \underline{b} \Rightarrow \underline{k}^u > k^n$ $\underline{K}^u > \frac{\mu \underline{\theta}}{\mathbb{E} \mathcal{L}}$ $\frac{F^{\underline{\sigma}}}{\mathbb{E}(\theta)}K^n$

so it can be possible that $K^n \geq \underline{K}^u > \frac{\mu \theta}{\mathbb{E}(\theta)} K^n$ when

$$
\underline{K}^{u} \leq K^{n} \Leftrightarrow K^{n} = \mathbb{E}(\theta) k^{n} \geq \mu \underline{\theta} k^{u}
$$
\n
$$
K^{n} = \mathbb{E}(\theta) \gamma (c \mathbb{E}(\theta)) \geq \underline{K}^{u} = \mu \underline{\theta} \gamma (c \underline{\theta} - \underline{b})
$$

Now

$$
G(0) = \overline{\theta} \gamma \left(c \overline{\theta} \right) > 0
$$

$$
G(1) = \underline{\theta} \gamma \left(c \underline{\theta} \right) < \underline{\theta} \gamma \left(c \underline{\theta} - \underline{b} \right)
$$

Then, it exists $\hat{\mu}: G(\hat{\mu}) = \hat{\mu} \underline{\theta} \gamma (c\underline{\theta} - \underline{b})$, such that $\underline{K}^u \leq K^n$ iff $\mu \leq \hat{\mu}$. So, whenever $\hat{\mu} > \mu^*$ when $\mu \in [\mu^*, \hat{\mu}]$, we have the result that $\Delta \Pi(\mu) \geq 0$ and $\underline{K}^u \leq K^n$. So, this completes the proof.